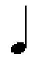
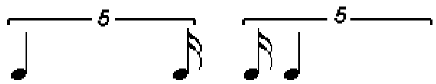
Table of Quintuplet Partitions

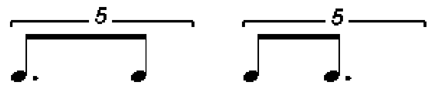
(a) 5



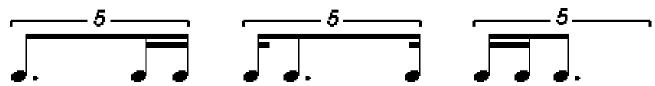
(b) 4+1



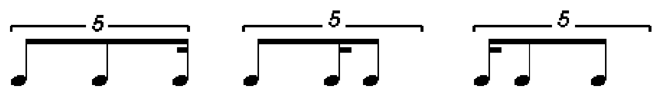
(c) 3+2



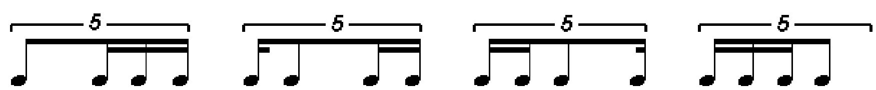
(d) 3+1+1



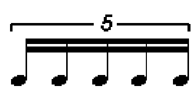
(e) 2+2+1



(f) 2+1+1+1



(g) 1+1+1+1+1



= 16 possibilities

Partitions of 5

The table above displays all possible “unique patterns" derivable within 1  = using only the , or whole multiples thereof, as the building block.

Patterns are classified vertically according to their component durations, as indicated at the left, where

5 = ; 4 = ; 3 = ; 2 = ; 1= .

Mathematicians will recognize these classifications as the partitions of the number 5.

The “partitions of a number” are all of the different ways in which the number *(n)* can be written, or thought of, as the sum of other (smaller) positive numbers.

In the particular case *(n =5)* there are 7 possible partitions (convention dictates that they be listed in this order):

(a) 5;
(b) 4+1;
(c) 3+2;
(d) 3+1+1;
(e) 2+2+1;
(f) 2+1+1+1;
(g) 1+1+1+1+1.

Arithmetically the order of the components of a specific partition does not matter – as an example 3+1+1 = 1+3+1 = 1+1+3 and they all = 5. Musically the specific ordering (or permutation) of the components is crucial, as in music it very much matters whether one plays long-short-short, or short-long-short, or short-short-long. These musically-imperative “unique patterns” are to be found on the horizontal.

Note that these “unique patterns” do not represent all possible permutations with every ordering preserved.

For example, consider lines d) or e), both of which have three objects (or notes) to be permuted. The total possible arrangements of any 3 objects is 3! (read as 3 factorial) which means

3 x 2 x 1 = 6;   i.e. if the objects are “abc” the 6 possible arrangements are: abc/acb/bac/bca/cab/cba.

However, on lines d) and e), two of the note values (or objects) are identical, and for those lines, it makes no practical difference (at least to musicians) which of the two identical values we rotate in to the first position, as either rotation provides the same pattern; hence, despite there being 3 objects to permute, there are not 6 *unique* possible arrangements but rather, only 3.

This may be clearer if we redo our “abc” example from above, using “aab”.

Now our 6 possible arrangements (using the same method of rotation as before) are:

aab/aba/aab/aba/baa/baa

from which it can be seen that there are only 3 unique patterns.

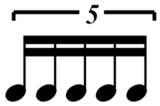
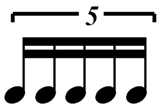
Similarly, for f), there are four objects = 4! = 4 x 3 x 2 x 1 = 24 possible arrangements, but as three of the objects are identical, permutation (or rotation) only produces four “unique patterns”.

Even though there are fewer “unique patterns” than there are total possible permutations, there are nevertheless more “unique patterns” than partitions – in the case of *(n =5)* there are 7 partitions, but 16 “unique patterns” – and it is the ever increasing number of “unique patterns” that helps cause our discomfort with rhythmic groupings greater than 4 (even plain old vanilla 32nd note groupings can be a pain – as can be seen from the difficulty one still has with the examples in Hindemith’s *Elementary Training* – where for the partitions of 8 there are simply so many “unique patterns” that we cannot be familiar with all of them).

The table can also be used for beats other than the ♩ by doubling or halving the internal values. By removing the quintuplet superscripts and brackets, the table can also be used as the basis for studying $\frac{5}{4}$, $\frac{5}{8}$, $\frac{5}{16}$.

For study purposes one should (probably) begin with the bottom classification, becoming comfortable saying, thinking, or tapping 1,2,3,4,5, many many times, at various speeds. Thereafter, one may proceed to the other patterns, always silently counting (while repeating each pattern) the internal 1,2,3,4,5, until one is quite comfortable grouping by 5, and no longer needs to make explicit the underlying 1,2,3,4,5, – anymore than one internally ferociously counts out 1,2,3,4, when playing or thinking  in one ♩.

Once comfortable with the patterns individually, it will prove of use to create random chains of patterns (at first repeating each pattern two or three times before moving to the next pattern, but eventually performing the entire chain without any consecutive repeats of a pattern).

If this table is being studied in a class situation, it will be useful to take a single pattern and divide it amongst two or more students, or even groups of students. One should start with the class in unison on  , and after the class has begun to feel comfortable with the trajectory of a unison  the teacher should distribute the notes so that (given five persons) each one gets to tap or say a single attack, trying to get the group to preserve the overall trajectory of phrase. This process should be used for as many different patterns as time will allow, as "hocket" is very important to learn and be comfortable with, as the passing around of a simple pattern between and amongst a number of instruments is a fairly common device in 20th century music.

For creating random chains of patterns, it might be enticing (in a class situation) to have the class generate the chains using the surrealist game "exquisite corpses" – i.e. the first student secretly chooses an arbitrary pattern, and writes it at the top of a piece of paper which is then folded, and passed to the next student in such a way that that student does not know what has been written. The second student then secretly chooses another arbitrary pattern and writes that down on the folded paper, folds the paper yet again, and passes it to the third student etc. etc. In this manner one would hope to achieve some unexpected progressions. This method may result in some patterns being repeated (when a student unknowingly chooses a pattern identical to the pattern previously just chosen) but this can either be left alone, or the unintentional repetition can be stricken. The entire class then taps/sings the entire final result, first in unison, and then eventually in hocket.

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